Scheme of Examination for M.Sc. Mathematics
w.e.f the session 2011-12

Semester – I

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-401	Advanced Abstract Algebra – I	80	20	100	3 Hours
MM-402	Real Analysis – I	80	20	100	3 Hours
MM-403	Topology	80	20	100	3 Hours
MM-404	Complex Analysis – I	80	20	100	3 Hours
MM-405	Differential Equations – I	80	20	100	3 Hours
MM-406	Practical-I	--	--	100	4 Hours

Semester – I

MM-401: Advanced Abstract Algebra-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

<u>Section – I (Two Questions)</u>

Automorphisms and Inner automorphisms of a group G. The groups Aut(G) and Inn(G). Automorphism group of a cyclic group. Normalizer and Centralizer of a non-empty subset of a group G. Conjugate elements and conjugacy classes. Class equation of a finite group G and its applications. Derived group (or a commutator subgroup) of a group G. perfect groups. Zassenhau's Lemma. Normal and Composition series of a group G. Scheier's refinement theorem. Jordan Holder theorem. Composition series of groups of order p^n and of Abelian groups. Caunchy theorem for finite groups. ∏ - groups and p-groups. Sylow ∏-subgroups and Sylow p-subgroups. Sylow's Ist, IInd and IIIrd theorems. Application of Sylow theory to groups of smaller orders.

<u>Section – II (Two Questions)</u>

Characteristic of a ring with unity. Prime fields Z/pZ and Q. Field extensions. Degree of an extension. Algebraic and transcendental elements. Simple field extensions. Minimal polynomial of an algebraic element. Conjugate elements. Algebraic extensions. Finitely generated algebraic extensions. Algebraic closure and algebraically closed fields. Splitting fields., finite fields.. Normal extensions.

<u>Section – III (Two Questions)</u>

Separable elements, separable polynomials and separable extensions. Theorem of primitive element. Perfect fields. Galois extensions. Galois group of an extension. Dedekind lemma Fundamental theorem of Galois theory. Frobenius automorphism of a finite field. Klein's 4-group and Diheadral group. Galois groups of polynomials. Fundamental theorem of Algebra.

Section – IV (Two Questions)

Solvable groups Derived series of a group G. Simplicity of the Alternating group A_n ($n \geq 5$). Non-solvability of the symmetric group Sn and the Alternating group An ($n \geq 5$). Roots of unity Cyclotomic polynomials and their irreducibility over Q Radicals extensions. Galois radical extensions. Cyclic extensions. Solvability of polynomials by radicals over Q. Symmetric functions and elementary symmetric functions. Construction with ruler and compass only.

Recommended Books:

1. I.D. Macdonald. :The theory of Groups
2. P.B. Bhattacharya
   S.K. Jain & S.R. Nagpal : Basic Abstract Algebra (Cambridge University Press 1995)

Reference Books:

1. Vivek Sahai and Vikas Bist : Algebra (Narosa publication House)
2. I.S. Luthar and I.B.S. Passi : Algebra Vol. 1 Groups (Narosa publication House)
3. I.N. Herstein : Topics in Algebra (Wiley Eastern Ltd.)
4. Surjit Singh and Quazi Zameeruddin : Modern Algebra (Vikas Publishing House 1990)

Semester-I

MM-402 : REAL ANALYSIS –I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Definition and existence of Riemann Stieltjes integral, properties of the integral, integration and differentiation, the fundamental theorem of integral calculus, integration by parts, integration of vector-valued functions, Rectifiable curves.
(Scope as in Chapter 6 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition).

Section-II (Two Questions)

Pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weirstrass M-test, Abel's test and Dirichlet's test for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann Stieltjes integration, uniform convergence and differentiation, existence of a real continuous nowhere differentiable function, equicontinous families of functions, Weierstrass approximation theorem (Scope as in Sections 7.1 to 7.27 of Chapter 7 of Principles of Mathematical Analysis by Walter Rudin, Third Edition).

Section-III (Two Questions)

Functions of several variables : linear transformations, Derivative in an open subset of R^n, Chain rule, Partial derivatives, directional derivatives, the contraction principle, inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Derivatives of higher order, mean value theorem for real functions of two variables, interchange of the order of differentiation, Differentiation of integrals.

(Scope as in relevant portions of Chapter 9 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition)

Section-IV (Two Questions)

Power Series : Uniqueness theorem for power series, Abel's and Tauber's theorem, Taylor's theorem, Exponential & Logarithm functions, Trigonometric functions, Fourier series, Gamma function
(Scope as in Chapter 8 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition).

Integration of differential forms: Partitions of unity, differential forms, stokes theorem

(scope as in relevant portions of Chapter 9 & 10 of 'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition).

Recommended Text:
'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition) McGraw-Hill, 1976.

Reference Books :

1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975.
3. A.J. White, Real Analysis; an introduction. Addison-Wesley Publishing Co., Inc., 1968.
4. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer, 1969.
5.Serge Lang, Analysis I & II, Addison-Wesley Publishing Company Inc., 1969.
M.Sc.(P)Mathematics Semester-I

Semester-I

MM-403: TOPOLOGY

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Definition and examples of topological spaces, Neighbourhoods, Neighbourhood system of a point and its properties, Interior point and interior of a set, interior as an operator and its properties, definition of a closed set as complement of an open set, limit point (accumulation point) of a set, derived set of a set, definition of closure of a set as union of the set and its derived set, Adherent point (Closure point) of a set , closure of a set as set of adherent (closure) points, properties of closure, closure as an operator and its properties, boundary of a set, Dense sets. A characterization of dense sets.
Base for a topology and its characterization, Base for Neighbourhood system, Sub-base for a topology.
Relative (induced) Topology and subspace of a topological space. Alternate methods of defining a topology using 'properties' of 'Neighbourhood system', 'Interior Operator', 'Closed sets', Kuratowski closure operator and 'base'.
First countable, Second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and a second countable space, Lindelof theorem.
Comparison of Topologies on a set, about intersection and union of topologies, infimum and supremum of a collection of topologies on a set, the collection of all topologies on a set as a complete lattice (scope as in theorems 1-16, chapter 1 of Kelley's book given at Sr. No. 1).

SECTION-II (Two Questions)

Definition, examples and characterisations of continuous functions, composition of continuous functions, Open and closed functions, Homeomorphism, embedding.
Tychonoff product topology in terms of standard (defining) subbase, projection maps, their continuity and openness, Characterisation of product topology as the smallest topology with projections continuous, continuity of a function from a space into a product of spaces.

T_0 , T_1, T_2,Regular and T_3 separation axioms, their characterization and basic properties i.e. hereditary property of T_0 , T_1, T_2, Regular and T_3 spaces, and productive property of T_1 and T_2 spaces.
Quotient topology w.r.t. a map, Continuity of function with domain a space having quotient topology, About Hausdorffness of quotient space (scope as in theorems 1, 2, 3, 5, 6, 8-11, Chapter 3 and relevant portion of chapter 4 of Kelley's book given at Sr.No.1)

Section-III (Two Questions)

Completely regular and Tychonoff ($T_{3\ 1/2}$), spaces, their hereditary and productive properties. Embedding lemma, Embedding theorem.
Normal and T_4 spaces : Definition and simple examples, Urysohn's Lemma, complete regularity of a regular normal space, T_4 implies Tychonoff, Tietze's extension theorem (Statement only). (Scope as in theorems 1-7, Chapter 4 of Kelley's book given at Sr. No. 1).
Definition and examples of filters on a set, Collection of all filters on a set as a p.o. set, finer filter, methods of generating filters/finer filters, Ultra filter (u.f.) and its characterizations, Ultra Filter Principle (UFP) i.e. Every filter is contained in an ultra filter. Image of filter under a function.
Convergence of filters: Limit point (Cluster point) and limit of a filter and relationship between them, Continuity in terms of convergence of filters. Hausdorffness and filter convergence.

Section-IV (Two Questions)

Compactness: Definition and examples of compact spaces, definition of a compact subset as a compact subspace, relation of open cover of a subset of a topological space in the sub-space with that in the main space, compactness in terms of finite intersection property (f.i.p.), continuity and compact sets, compactness and separation properties, Closedness of compact subset, closeness of continuous map from a compact space into a Hausdorff space and its consequence, Regularity and normality of a compact Hausdorff space.
Compactness and filter convergence, Convergence of filters in a product space, compactness and product space. Tychonoff product theorem using filters, Tychonoff space as a subspace of a compact Hausdorff space and its converse, compactification and Hausdorff compactification, Stone-Cech compactification, (Scope as in theorems 1,7-11, 13, 14, 15, 22-24, Chapter 5 of Kelley's book given at Sr. No. 1).

Books :

1. Kelley, J.L. : General Topology.
2. Munkres, J.R. : Topology, Second Edition, Prentice Hall of India/ Pearson.

Semester-I

MM-404: COMPLEX ANALYSIS-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Power series, its convergence, radius of convergence, examples, sum and product, differentiability of sum function of power series, property of a differentiable function with derivative zero. expz and its properties, logz, power of a complex number (z), their branches with analyticity.
Path in a region, smooth path, p.w. smooth path, contour, simply connected region, multiply connected region, bounded variation, total variation, complex integration, Cauchy-Goursat theorem, Cauchy theorem for simply and multiply connected domains.

Section II (Two Questions)

Index or winding number of a closed curve with simple properties. Cauchy integral formula. Extension of Cauchy integral formula for multiple connected domain. Higher order derivative of Cauchy integral formula. Gauss mean value theorem Morera's theorem. Cauchy's inequality. Zeros of an analytic function, entire function, radius of convergence of an entire function, Liouville's theorem, Fundamental theorem of algebra, Taylor's theorem.

Section-III (Two Questions)

Maximum modulus principle, Minimum modulus principle. Schwarz Lemma. Singularity, their classification, pole of a function and its order. Laurent series, Cassorati – Weiertrass theorem Meromorphic functions, Poles and zeros of Meromorphic functions. The argument principle, Rouche's theorem, inverse function theorem.

Section-IV (Two Questions)

Residue : Residue at a singularity, residue at a simple pole, residue at infinity. Cauchy residue theorem and its use to calculate certain integrals, definite integral ($\int_0^{2\Pi}$ f(cosθ, sinθ) dθ, $\int_{-\infty}^{\infty}$f(x)dx), integral of the type $\int_0^{\infty}$f(x) sinmx dx or $\int_0^{\infty}$ f(x) cosmx dx, poles on the real axis, integral of many valued functions.

Bilinear transformation, their properties and classification, cross ration, preservance of cross ration under bilinear transformation, preservance of circle and straight line under bilinear transformation, fixed point bilinear transformation, normal form of a bilinear transformation. Definition and examples of conformal mapping, critical points.

Books recommended :

1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
2. Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.
3. Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

Reference Books :

1. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
2. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
3. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
4. Mark J.Ablewitz and A.S.Fokas, Complex Variables : Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
5. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
6. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

Semester-I

MM-405: Differential Equations –I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section –I (Two Questions)

Preliminaries: Initial value problem and equivalent integral equation, ε-approximate solution, equicontinuous set of functions.
Basic theorems: Ascoli- Arzela theorem, Cauchy –Peano existence theorem and its corollary. Lipschitz condition. Differential inequalities and uniqueness, Gronwall's inequality. Successive approximations. Picard-Lindelöf theorem. Continuation of solution, Maximal interval of existence, Extension theorem. Kneser's theorem (statement only)
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Section-II (Two Questions)

Linear differential systems: Definitions and notations. Linear homogeneous systems; Fundamental matrix, Adjoint systems, reduction to smaller homogeneous systems. Non-homogeneous linear systems; variation of constants. Linear systems with constant coefficients. Linear systems with periodic coefficients; Floquet theory.
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Section-III (Two Questions)

Higher order equations: Linear differential equation (LDE) of order n; Linear combinations, Linear dependence and linear independence of solutions. Wronskian theory: Definition, necessary and sufficient condition for linear dependence and linear independence of solutions of homogeneous LDE. Abel's Identity, Fundamental set, More Wronskian theory. Reduction of order. Non-homogeneous LDE. Variation of parameters. Adjoint equations, Lagrange's Identity, Green's formula. Linear equation of order n with constant coefficients. (Relevant portions from the books of 'Theory of Ordinary Differential Equations' by Coddington and Levinson and the book 'Differential Equations' by S.L. Ross)

Section –IV (Two Questions)

System of differential equations, the n-th order equation. Dependence of solutions on initial conditions and parameters: Preliminaries, continuity and differentiability.
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Maximal and Minimal solutions. Differential inequalities. A theorem of Wintner. Uniqueness theorems: Kamke's theorem, Nagumo's theorem and Osgood theorem.
(Relevant portions from the book 'Ordinary Differential Equations' by P. Hartman)

Referneces:

1. E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw-Hill , 2000.
2. S.L. Ross, *Differential Equations*, John Wiley & Sons,
3. P. Hartman, *Ordinary Differential Equations*, John Wiley & Sons NY, 1971.
4. G. Birkhoff and G.C. Rota, *Ordinary Differential Equations*, John Wiley & Sons, 1978.
5. G.F. Simmons, *Differential Equations*, Tata McGraw-Hill , 1993.
6. I.G. Petrovski, *Ordinary Differential Equations*, Prentice-Hall, 1966.
7. D. Somasundaram, *Ordinary Differential Equations, A first Course*, Narosa Pub., 2001.
8. S.G. Deo, V. Lakshmikantham and V. Raghavendra, *Textbook of Ordinary Differential Equations*, Tata McGraw-Hill , 2006.

Semester-I

Paper MM-406 : Practical-I

Examination Hours : 4 hours
Max. Marks : 100

Part-A : Problem Solving

In this part, problem-solving techniques based on papers MM-401 to MM-405 will be taught.

Part-B : Implementation of the following programs in ANSI C.

1. Use of nested **if.. .else** in finding the smallest of four numbers.
2. Use series sum to compute **sin(x)** and **cos(x)** for given angle **x** in degrees. Then, check error in verifying $\sin^2 x+\cos^2(x)=1$.
3. Verify $\sum n^3 =\{\sum n\}^2$, (where n=1,2,..,m) & check that prefix and postfix increment operator gives the same result.
4. Compute simple interest of a given amount for the annual rate = .12 if amount >=10,000/- or time >=5 years; =.15 if amount >=10,000/- and time >=5 years; and = .10 otherwise.
5. Use array of pointers for alphabetic sorting of given list of English words.
6. Program for interchange of two rows or two columns of a matrix. Read/write input/output matrix from/to a file.
7. Calculate the eigenvalues and eigenvectors of a given symmetric matrix of order 3.
8. Calculate standard deviation for a set of values {x(j)j=l,2,…,n} having the corresponding frequencies {f(j)j=l,2,…,n}.
9. Find GCD of two positive integer values using pointer to a pointer.
10. Compute GCD of 2 positive integer values using recursion.
11. Check a given square matrix for its positive definite form.
12. To find the inverse of a given non-singular square matrix.

Note :- Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.

<u>Semester – II</u>

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-407	Advanced Abstract Algebra – II	80	20	100	3 Hours
MM-408	Real Analysis – II	80	20	100	3 Hours
MM-409	Computer Programming (Theory)	80	20	100	3 Hours
MM-410	Complex Analysis – II	80	20	100	3 Hours
MM-411	Differential Equations – II	80	20	100	3 Hours
MM-412	Practical-II	--	--	100	4 Hours

Semester – II

MM-407: Advanced Abstract Algebra-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

<u>Section-I (Two Questions)</u>

Commutators and higher commutators. Commutators identities. Commutator subgroups. Derived group. Three subgroups Lemma of P.Hall. Central series of a group G. Nilpotent groups. Centre of a nilpotent group. Subgroups and factor subgroups of nilpotent groups. Finite nilpotent groups. Upper and lower central series of a group G and their properties. Subgroups of finitely generated nilpotent groups. Sylow-subgroups of nilpotent groups.
(Scope of the course as given in the book at Sr. No. 2).

<u>Section-II (Two Questions)</u>

Similar linear transformations. Invariant subspaces of vector spaces. Reduction of a linear transformation to triangular form. Nilpotent transformations. Index of nilpotency of a nilpotent transformation. Cyclic subspace with respect to a nilpotent transformation. Uniqueness of the invariants of a nilpotent transformation.

Primary decomposition theorem. Jordan blocks and Jordan canonical forms. Cyclic module relative to a linear transformation. Companion matrix of a polynomial f(x). Rational Canonicals form of a linear transformation and its elementary divisior. Uniqueness of the elementary divisior. (Sections 6.4 to 6.7 of the book. Topics in Algebra by I.N. Herstein).

<u>Section-III (Two Questions)</u>

Modules, submodules and quotient modules. Module generated by a non-empty subset of an R-module. Finitely generated modules and cyclic modules. Idempotents. Homomorphism of R-modules. Fundamental theorem of homomorphism of R-modules. Direct sum of modules. Endomorphism rings $End_Z(M)$ and $End_R(M)$ of a left R-module M. Simple modules and completely reducible modules (semi-simple modules). Finitely generated free modules. Rank of a finitely generated free module. Submodules of free modules of finite rank over a PID. (Sections 14.1 to 14.5 of the book Basic Abstract Algebra by P.B. Bhattacharya S.K. Jain and S.R. Nagpal)

Section-IV (Two Questions)

Endomorphism ring of a finite direct sum of modules. Finitely generated modules. Ascending and descending chains of sub modules of an R-module. Ascending and Descending change conditions (A.C.C. and D.C.C.). Noetherian modules and Noetherian rings. Finitely co-generated modules. Artinian modules and Artinian rings. Nil and nilpotent ideals. Hilbert Basis Theorem. Structure theorem of finite Boolean rings. Wedeerburn-Artin theorem and its consequences. (sections 19.1 to 19.3 of the book Basic Abstract Algebra by P.B. Bhattacharya S.K. Jain and S.R. Nagpal).

Recommended Books:

1. Basic Abstract Algebra	:	P.B. Bhattacharya S.R. Jain and S.R. Nagpal
2. Theory of Groups	:	I.D. Macdonald
3. Topics in Algebra	:	I.N. Herstein
4. Group Theory	:	W.R. Scott

Semester-II

MM-408 : REAL ANALYSIS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Lebesgue outer measure, elementary properties of outer measure, Measurable sets and their properties, Lebesgue measure of sets of real numbers, algebra of measurable sets, Borel sets and their measurability, characterization of measurable sets in terms of open, closed, F and G sets, existence of a non-measurable set.

Lebesgue measurable functions and their properties, characteristic functions, simple functions, approximation of measurable functions by sequences of simple functions, measurable functions as nearly continuous functions. Borel measurability of a function.

Section-II (Two Questions)

Almost uniform convergence, Egoroff's theorem, Lusin's theorem, convergence in measure, F.Riesz theorem that every sequence which is convergent in measure has an almost everywhere convergent subsequence.

The Lebesgue Integral :

Shortcomings of Riemann integral, Lebesgue integral of a bounded function over a set of finite measure and its properties, Lebsegue integral as a generalization of the Riemann integral, Bounded convergence theorem, Lebesgue theorem regarding points of discontinuities of Riemann integrable functions.

Section-III (Two Questions)

Integral of a non negative function, Fatou's lemma, Monotone convergence theorem, integration of series, the general Lebesgue integral, Lebesgue convergence theorem.

Differentiation and Integration :

Differentiation of monotone functions, Vitali's covering lemma, the four Dini derivatives, Lebesgue differentiation theorem, functions of bounded variation and their representation as difference of monotone functions.

Section-IV (Two Questions)

Differentiation of an integral, absolutely continuous functions, convex functions, Jensen's inequality.

The L^p spaces

The L^p spaces, Minkowski and Holder inequalities, completeness of L^p spaces, Bounded linear functionals on the L^p spaces, Riesz representation theorem.

Recommeded Text :

'Real Analysis' by H.L.Royden (3^{rd} Edition) Prentice Hall of India, 1999.

Reference Books :

1. G.de Barra, Measure theory and integration, Willey Eastern Ltd.,1981.
2. P.R.Halmos, Measure Theory, Van Nostrans, Princeton, 1950.
3. I.P.Natanson, Theory of functions of a real variable, Vol. I, Frederick Ungar Publishing Co., 1961.
4. R.G.Bartle, The elements of integration, John Wiley & Sons, Inc.New York, 1966.
5. K.R.Parthsarthy, Introduction to Probability and measure, Macmillan Company of India Ltd.,Delhi, 1977.

   P.K.Jain and V.P.Gupta, Lebesgue measure and integration, New age International (P) Ltd., Publishers, New Delhi, 1986.

Semester-II

MM-409 : Computer Programming (Theory)

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

<u>Section-I (Two Questions)</u>

Numerical constants and variables; arithmetic expressions; input/output; conditional flow; looping.

<u>Section-II (Two Questions)</u>

Logical expressions and control flow; functions; subroutines; arrays.

<u>Section- III(Two Questions)</u>

Format specifications; strings; array arguments, derived data types.

<u>Section- IV(Two Questions)</u>

Processing files; pointers; modules; FORTRAN 90 features; FORTRAN 95 features.

Recommended Text:

V. Rajaraman : Computer Programming in FORTRAN 90 and 95; Printice-Hall of India Pvt. Ltd., New Delhi, 1997.

References :

1. V. Rajaraman : Computer Programming in FORTRAN 77, Printice-Hall of India Pvt. Ltd., New Delhi, 1984.
2. J.F. Kerrigan : Migrating of FORTRAN 90, Orielly Associates, CA, USA, 1993.
3. M.Metcalf and J.Reid : FORTRAN 90/95 Explained, OUP, Oxford, UK, 1996.

Semester-II

MM-410 : COMPLEX ANALYSIS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Spaces of analytic functions and their completeness, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem, infinite products, Weierstrass factorization theorem, Factorization of sine function, Gamma function and its properties, functional equation for gamma function, Integral version of gamma function.

Section- II (Two Questions)

Reimann-zeta function, Riemann's functional equation, Runge's theorem, Mittag-Leffler's theorem.
Analytic continuation, uniqueness of direct analytic continuation, uniqueness of analytic continuation along a curve, Power series method of analytic continuation , Schwarz reflection principle.

Section –III (Two Questions)

Monodromy theorem and its consequences. Harmonic function as a disk, Poisson's Kernel. Harnack's inequality, Harnack's theorem, Canonical product, Jensen's formula, Poisson-Jensen formula, Hadamard's three circle theorem. Dirichlet problem for a unit disk. Dirichlet problem for a region, Green's function.

Section –IV (Two Questions)

Order of an entire function, Exponent of convergence, Borel theorem, Hadamard's factorization theorem. The range of an analytic function, Bloch's theorem, Little-Picard theorem, Schottky's theorem, Montel-Carathedory theorem, Great Picard theorem.
Univalent functions, Bieberbach's conjecture (Statement only), and $1^7/ 4$ theorem.

Books recommended :

1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
2. Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.
3. Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

Reference Books :

1. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
2. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
3. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
4. Mark J.Ablewitz and A.S.Fokas, Complex Variables : Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
5. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
6. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

Semester-II

MM-411: DIFFERENTIAL EQUATIONS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all, taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section –I (Two Questions)

Linear second order equations: Preliminaries, self adjoint equation of second order, Basic facts, superposition principle, Riccati's equation, Prüffer transformation, zero of a solution, Oscillatory and non-oscillatory equations. Abel's formula. Common zeros of solutions and their linear dependence.
(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Section –II (Two Questions)

Sturm theory: Sturm separation theorem, Sturm fundamental comparison theorem and its corollaries. Elementary linear oscillations.
Autonomous systems: the phase plane, paths and critical points, Types of critical points; Node, Center, Saddle point, Spiral point. Stability of critical points. Critical points and paths of linear systems: basic theorems and their applications.
(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Section-III (Two Questions)

Critical points and paths of non-linear systems: basic theorems and their applications. Liapunov function. Liapunov's direct method for stability of critical points of non-linear systems.
Limit cycles and periodic solutions: Limit cycle, existence and non-existence of limit cycles, Benedixson's non-existence criterion. Half-path or Semiorbit, Limit set, Poincare-Benedixson theorem. Index of a critical point.

(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

<u>Section-IV (Two Questions)</u>

Second order boundary value problems(BVP): Linear problems; periodic boundary conditions, regular linear BVP, singular linear BVP; non-linear BVP. Sturm-Liouville BVP: definitions, eigen value and eigen function. Orthogonality of functions, orthogonality of eigen functions corresponding to distinct eigen values. Green's function. Applications of boundary value problems. Use of Implicit function theorem and Fixed point theorems for periodic solutions of linear and non-linear equations.
(Relevant portions from the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Referneces:

1. E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw-Hill , 2000.
2. S.L. Ross, *Differential Equations*, John Wiley & Sons,
3. S.G. Deo, V. Lakshmikantham and V. Raghavendra, *Textbook of Ordinary Differential Equations*, Tata McGraw-Hill , 2006.
4. P. Hartman, *Ordinary Differential Equations*, John Wiley & Sons NY, 1971.
5. G. Birkhoff and G.C. Rota, *Ordinary Differential Equations*, John Wiley & Sons, 1978.
6. G.F. Simmons, *Differential Equations*, Tata McGraw-Hill , 1993.
7. I.G. Petrovski, *Ordinary Differential Equations*, Prentice-Hall, 1966.
8. D. Somasundaram, *Ordinary Differential Equations, A first Course*, Narosa Pub., 2001.

Semester-II

Paper MM-412 : Practical-II

Examination Hours : 4 hours
Max. Marks : 100

Part-A : Problem Solving

In this part, problem solving techniques based on papers MM-407 to MM-411 will be taught.

Part-B : Implementation of the following programs in FORTRAN-90

1. Calculate the area of a triangle with given lengths of its sides.
2. Given the centre and a point on the boundary of a circle, find its perimeter and area.
3. To check an equation $ax^2 + by^2 + 2cx + 2dy + e = 0$ in (x, y) plane with given coefficients for representing parabola/ hyperbola/ ellipse/ circle or else.
4. For two given values x and y, verify g*g=a*h, where a, g and h denote the arithmetic, geometric and harmonic means respectively.
5. Use IF..THEN…ELSE to find the largest among three given real values.
6. To solve a quadratic equation with given coefficients, without using COMPLEX data type.
7. To find the location of a given point (x,y) i) at origin, ii) on x-axis or y-axis iii) in quadrant I, II, III or IV.
8. To find if a given 4-digit year is a leap year or not.
9. To find the greatest common divisor (gcd) of two given positive integers.
10. To verify that sum of cubes of first m positive integers is same as the square of the sum of these integers.
11. Find error in verifying sin(x+y)= sin(x) cos(y)+cos(x)sin(y), by approximating the sin(x) and cos(x) functions from the finite number of terms in their series expansions.
12. Use SELECT…CASE to calculate the income tax on a given income at the existing rates.

Note :- Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.